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Section 4.3 Solving Trigonometric Functions with Horizontal compressions and factoring:

1. Solve the following trigonometric equations for θ; $0 \le \theta \le 360°$ by factoring:

a) $\sin x \cos x = 0$

either $\sin x = 0$ or $\cos x = 0$

$x_1 = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

b) $(\tan x - 1)(4\sin^2 x - 3) = 0$

$\tan x - 1 = 0$ $\quad 4\sin^2 x - 3 = 0$

$\tan x = 1$ $\quad \sin x = \pm\sqrt{\frac{3}{4}}$

$x = \tan^{-1} 1$ $\quad x = \sin^{-1}\left(\pm \frac{\sqrt{3}}{2}\right)$

$x_1 = 45°$

$x_2 = 225°$

$x = 60°, 120°, 240°, 300°$

c) $\sin^2 x + \sin x - 2 = 0$

$\sin x = A$

$A^2 + A - 2 = 0$

$(A - 1)(A + 2) = 0$

$\sin x = 1$ $\quad$ ~~$\sin x = -2$~~ reject

$x = 90°$

d) $2\sin\theta - \csc\theta = 1$

$2\sin\theta - \frac{1}{\sin\theta} = 1$

$2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{2}$ $\quad \sin\theta = 1$

$\theta = 210°, 330°$ $\quad \theta_1 = 90°$

e) $2\sec\theta = \tan\theta + \cot\theta$

$2\frac{1}{\cos\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$2\sin\theta = \sin^2\theta + \cos^2\theta$

$= 1$

$\sin\theta = \frac{1}{2}$

$\theta = 30°, 150°$

e) $\tan\theta + 3\cot\theta = 4$

$\tan\theta + \frac{3}{\tan\theta} = 4$

$\tan^2\theta + 3 = 4\tan\theta$

$\tan^2\theta - 4\tan\theta + 3 = 0$

$(\tan\theta - 1)(\tan\theta - 3) = 0$

$\theta = \tan^{-1}(1) = 45°, 225°$ $\quad \theta = \tan^{-1}(3) = 71.6°, 251.6°$

f) $\cos\theta - \sqrt{3}\sin\theta = 1$

$\cos\theta - \sqrt{3}\cdot\sqrt{1 - \cos^2\theta} = 1$

$\sqrt{3 - 3\cos^2\theta} = \cos\theta - 1$

$3 - 3\cos^2\theta = \cos^2\theta - 2\cos\theta + 1$

$4\cos^2\theta - 2\cos\theta - 2 = 0$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

$\theta = 120°, 240°$ reject

$\theta = \cos^{-1}(1)$

$\theta = 0, 360°$

g) $2\cos\theta = 1 - \sin\theta$

$2\sqrt{1 - \sin^2\theta} = 1 - \sin\theta$

$4(1 - \sin^2\theta) = 1 - 2\sin\theta + \sin^2\theta$

$4 - 4\sin^2\theta = 1 - 2\sin\theta + \sin^2\theta$

$5\sin^2\theta - 2\sin\theta - 3 = 0$

$(5\sin\theta + 3)(\sin\theta - 1) = 0$

$\theta = \sin^{-1}\left(-\frac{3}{5}\right)$ $\quad \theta = \sin^{-1}(1)$

$\theta = -36.87°, 216.87°$ reject $\quad \theta = 90°$

PLUG ANSWERS BACK IN! extraneous

h) $\cos\theta + \cos 2\theta = 0$

$\cos\theta + \cos^2\theta - \sin^2\theta = 0$

$\cos\theta + \cos^2\theta = 1 - \cos^2\theta$

$2\cos^2\theta + \cos\theta - 1 = 0$

$\frac{1}{2}$ ——— -1

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\theta_1 = \cos^{-1}\left(\frac{1}{2}\right)$ $\theta = \cos^{-1}(-1)$

$\theta = 60°, 300°$ $\theta = 180°$

j) $\sin 2\theta + \sin\theta = 0$

$2\sin\theta\cos\theta + \sin\theta = 0$

$\sin\theta(2\cos\theta + 1) = 0$

$\sin\theta = 0$ $2\cos\theta + 1 = 0$

$\theta_1 = 0, 180°, 360°$ $\theta_2 = \cos^{-1}\left(-\frac{1}{2}\right)$

$\theta = 120°, 240°$

i) $2\sin\theta + \csc\theta = 3$

$2\sin\theta + \frac{1}{\sin\theta} = 3$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

$\frac{1}{2}$ ——— -1

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$\theta_1 = \sin^{-1}\left(\frac{1}{2}\right)$ $\theta_2 = \sin^{-1}(1)$

$\theta = 30°, 150°$ $\theta = 90°$

k) $2\tan\theta\sin\theta - \tan\theta = 0$

$\tan\theta(2\sin\theta - 1) = 0$

$\tan\theta = 0$ $2\sin\theta - 1 = 0$

$\theta = \tan^{-1}(0)$ $\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = 0°, 180°, 360°$ $\theta = 30°, 150°$

2. Solve the following trigonometric equations with horizontal compression for θ; $0 \le \theta \le 360°$

$\sin 4\theta = \frac{\sqrt{3}}{2}$

$4\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$4\theta = 60°, 120°$

Period: $\frac{2\pi}{4} = \frac{\pi}{2} = 90°$

$\theta = 15°, 105°, 195°, 285°, 120°, 210°, 300°$

$(2\cos 3\theta - 1)(4\tan 2\theta - 3) = 0$

$2\cos 3\theta - 1 = 0$ $4\tan 2\theta - 3 = 0$

$3\theta = \cos^{-1}\left(\frac{1}{2}\right)$ $2\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$3\theta = 60°, 300°$

$\theta = 20°, 140°, 260°, 100°, 220°, 340°$

Period: $\frac{2\pi}{3} = 120°$ Period: $\frac{\pi}{2} = 90°$

$\theta = 18.4°, 108.4°, 197.4°, 288.4°$

$2\sin^2 2\theta - \sin 2\theta - 1 = 0$

$\frac{1}{2}$ ——— -1

$(2\sin 2\theta + 1)(\sin 2\theta - 1) = 0$

$2\theta = \sin^{-1}\left(-\frac{1}{2}\right)$ $2\theta = \sin^{-1}(1)$

$2\theta = 210°, 330°$ $2\theta = 90°$

Period: $\frac{2\pi}{2} = 180°$

$\theta = 105°, 165°, 285°, 345°$ $\theta = 45°, 225°$

$\sin 2\theta\cos 2\theta\tan 2\theta = 0$

$\sin 2\theta = 0$ $\cos 2\theta = 0$

$2\theta = \sin^{-1}(0)$ $2\theta = \cos^{-1}(0)$

$2\theta = 0°, 180°, 360°$ $2\theta = 90°, 270°$

$\theta = 0, 90, 180, 270, 360°$ $\theta = 45°, 135°, 225°, 315°,$

Period: 180°

$\tan 2\theta = 0$

$2\theta = \tan^{-1}(0)$

$2\theta = 0, 180°$

Period: 90°

0, 90, 180, 270, 360

3. How many solutions will the equation $\cos 3\theta = 0.5$ for $0° \le \theta \le 360°$. Solve for θ.

$3\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$3\theta = 60°, 300°$

$\theta = 20°, 140°, 260°$
$100°, 220°, 340°$

Period: $\frac{2\pi}{3} = 120°$

4. Solve for θ, for $0° < \theta < 360°$: $\sin^2 2\theta - \cos^2 2\theta = 0$

$1 - \cos^2 2\theta - \cos^2 2\theta = 0$

$2\cos^2 2\theta = 1$

$\cos^2 2\theta = \frac{1}{2}$

$\cos 2\theta = \pm\sqrt{\frac{1}{2}}$

$2\theta = \cos^{-1}\left(\pm\sqrt{\frac{1}{2}}\right)$

$2\theta = 45°, 135°, 225°, 315°$

$\theta = \frac{45}{2}, \frac{135}{2}, \frac{225}{2}, \frac{315}{2}$
$\frac{405}{2}, \frac{495}{2}, \frac{585}{2}, \frac{675}{2}$

$2\pi \to 180°$

Period: 180°

5. Solve for θ, for $0° < \theta < 360°$: $\sin^3 2\theta + 3\sin^2 2\theta + 3\sin 2\theta = 0$

Let $\sin 2\theta = A$ $\quad A^3 + 3A^2 + 3A = 0$

$A(A^2 + 3A + 3) = 0$

$A = 0$ or $A = \frac{-3 \pm \sqrt{9 - 12}}{2} = \emptyset$

$\sin 2\theta = 0$

$2\theta = \sin^{-1}(0)$

$2\theta = 0, 180, 360$ Period: 180°

$\theta = 0, 90, 180, 270°, 360°$

6. Solve for θ, for $0° < \theta < 360°$: $(\sin 2\theta - \cos 2\theta)(1 + \sin\theta\cos\theta) = 0$

$\sin 2\theta - \cos 2\theta = 0$ or $1 + \frac{\sin 2\theta}{2} = 0$

$\sin 2\theta = \sqrt{1 - \sin^2 2\theta}$

$\sin 2\theta = -2$

$2\theta = \sin^{-1}(-2)$

rej. impossible

$\sin^2 2\theta = 1 - \sin^2 2\theta$

$2\sin^2 2\theta = 1$

$\sin^2 2\theta = \frac{1}{2}$

$2\theta = \sin^{-1}\left(\pm\sqrt{\frac{1}{2}}\right)$

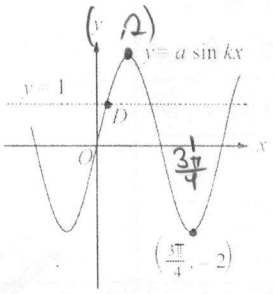
$\theta = 45°, 135°, 225°, 315°$ all rej

$\theta = \frac{45}{2}, \frac{135}{2}, \frac{225}{2}, \frac{315}{2}, \frac{405}{2}, \frac{495}{2}, \frac{585}{2}, \frac{675}{2}$ all rej

Period: $\frac{360}{2}$

OR turn to tan function. (No extraneous roots)

7. The graph of the equation $y = a \sin kx$ is shown in the diagram, and the point $\left(\frac{3\pi}{4}, -2\right)$ is the minimum point indicated. The line $y = 1$ intersects the graph at point D. What are the coordinates of D?



Period = π

$k = B = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$

$y = 2\sin(2x) = 1$

$\sin 2x = \frac{1}{2}$

$2x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$x = \frac{\pi}{12}$

(a) Determine all angles θ with $0° \le \theta \le 180°$ and $\sin^2\theta + 2\cos^2\theta = \frac{7}{4}$.

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cos^2\theta = \frac{7}{4}$

$\cos^2\theta = \frac{3}{4}$

$\theta = \cos^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$

$\theta = 30°, 150°$